# Chapter 64
# Doppler-Aided Algorithm for BeiDou Position

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**Abstract** Real-time GNSS positioning rely mainly on pseudorange and carrier-phase. The pseudorange is not accurate because of a lot of noise and interference in environment. High-precision carrier-phase used to smooth pseudorange. While, when cycle slip occurs, the carrier-phase measurement is bias. Unlike Carrier observations, the Doppler observations tend to have higher continuity. In Beidou system, GEO satellites local in geostationary orbit with lower dynamic. So, we can use the small noise bandwidth to improve the accuracy of measurement. This paper analyzes the characteristics and methods of Doppler-assisted positioning the the settlement equation using the weighted least squares method. The simulation and test results show that the positioning of the reliability of the results has been improved.

**Keywords** Doppler-aided · BeiDou System · Position Algorithm · Least square

## 64.1 Introduction

Due to satellite signal need to go through the long distance transmission, the transmission proceeds is interference. The interference is including thermal noise, ionospheric scintillation, tropospheric delay and multipath. Those interferences

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make the pseudorange measurement inaccurate. While the precision of code measurement is lower by 2–3 magnitudes than that of carrier, the use of carrier phase smoothing pseudorange is a frequently method. Hatch filter [1] using carrier phase difference as an accurate measurement to smooth pseudorange, is widely used. However, in some circumstances, such as the trees obscured or low signal-to-noise ratio, the phase-locked loop (PLL) may loss of lock, which causes resulting in a discontinuity of the carrier phase measurements. If the loss of PLL lock cannot be detected, the positioning results are biased. Hatch filter need to set a period, to prevent the errors from accumulating. Cycle slips occurring frequently, results the Hatch filter performance is not prominent in urban environments.

Doppler measurement is good at continuity and anti-jamming. Doppler measurements to assist pseudorange smoothing become a new choice [2]. In the following sections, firstly brief overview of BeiDou signal structure and satellite constellation is described. Then Doppler aided algorithm is introduced and PLL noise bandwidth is analysis. Finally numerical test results are shown and conclusion is given.

## 64.2 BeiDou Signal Model and Satellite Constellation

### *64.2.1 BeiDou Broadcast Signal*

The B1 signal radio (channel I) from BeiDou satellite [3] can be given as:

$$S_{B1i}(t) = \sqrt{2P_c}D_i(t - t_i)C_i(t - t_i)\cos[2\pi f_i(t - t_i) + \theta_i] + n_i(t) \qquad (64.1)$$

where: i is the satellite number, $S_{B1i}$ is the B1 signal received from satellite i, $P_c$ is the ranging code power, $D_i(t)$ is the navigation data, $f_i$ is carrier frequency, $C_i(t)$ is the pseudorandom noise spreading sequence ranging code, $t_i$ is the propagation delay, $\theta_i$ is unknown phase, and $n_i(t)$ is noise.

From formula (64.1), the signal transmitting delay can be got from the difference of code or carrier measurements. When the code delay measurements are used to determine pseudorange, the pseudorange is expressed as:

$$\rho_i(t) = r_i + c\delta t(t) + I_i(t) + T_i(t) + \varepsilon_{p,i}(t) \qquad (64.2)$$

where $\rho_i(t)$ is the pseudorange of satellite i, $c$ is velocity of light, $\delta t(t)$ is the clock error, $I_i(t)$ is ionosphere delay, $T_i(t)$ is troposphere delay, and $\varepsilon_{p,i}(t)$ is the measurement noise. $r_i$ is the geometric distance between satellite i and receiver ($x_i, y_i, z_i$ is satellites position, $x, y, z$ is receiver position):

$$r_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \qquad (64.3)$$

The formula (64.3) is substituted into (64.2), and rearrange, we can get the position formula:

$$(\rho_i - I_i - T_i) = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + c\delta t + \varepsilon_{p,i} \tag{64.4}$$

The satellite position is solved from satellite broadcast message. The ionospheric and tropospheric delay is modeled.

### *64.2.2 BeiDou Satellite Constellations*

Beidou Navigation Satellite System is an independent system offering regional position and timing service now. Comparing with GPS satellite, BeiDou navigation system consists of three kinds of orbit satellites, include Geostationary Earth Orbit (GEO) satellites, Medium Earth Orbit (MEO) satellites and Inclined Geosynchronous Satellite Orbit (IGSO) satellites.

The GEO satellites are local in orbit with low dynamic. The elevation change of visible satellites shown in Fig. 64.1 in 24 h (cut-off angle 10°). The elevations of GEO satellites only have minor changes. Unlike GPS satellite, BeiDou receiver can decrease the PLL noise bandwidth for low dynamic in order to reduce noise.

## 64.3 Doppler-Aided Position Algorithm

### *64.3.1 Doppler Model*

Doppler is generated by the relative movement of the satellite and user. According to General relativity, the doppler is expressed as:

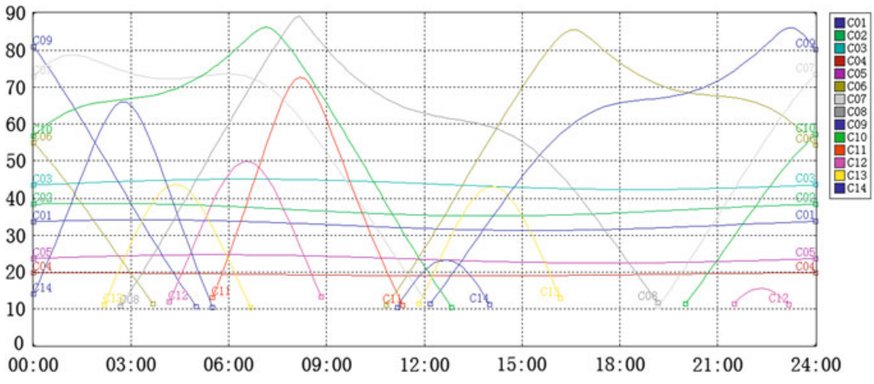


**Fig. 64.1** Elevation change of visible satellites of Beijing (24 h)

$$\frac{f_r}{f_0} = \frac{1 - \frac{(\vec{v}_i - \vec{v})(\vec{p}_i - \vec{p})}{c\|\vec{p}_i - \vec{p}\|}}{\sqrt{1 - \frac{\|\vec{v}_i - \vec{v}\|^2}{c^2}}} \tag{64.5}$$

where $f_r$ is carrier frequency at the receiver, $f_0$ is the transmitted carrier frequency (1561.098 MHz), $\vec{p}_i, \vec{v}_i$ is the vector of satellite position and velocity. $\vec{p}, \vec{v}$ is the vector of user position and velocity.

The relative movement between satellites and user is lower than the velocity of light. The denominator of formula (64.5) is close to 1, then the equation simplifies as:

$$f_{di} = \frac{f_0}{c} * \frac{(\vec{p}_i - \vec{p})}{\|\vec{p}_i - \vec{p}\|} * (\vec{v}_i - \vec{v}) \tag{64.6}$$

Equation (64.6) consists of three parts: the first part is constant, and the second part represents the unit vector from satellite to user, and the last part is relative velocity vector.

## 64.3.2 Position Algorithm Using Pseudorange

The least square (LS) method is used to calculate the location. The equation include four unknowns $(x, y, z, \delta t)$ is settled with four satellites.

The least squares method can only solving the linear equation. The nonlinear Eq. (64.4) are linearized with Talyor series [4].

$$\rho_i^0 - (\rho_i - I_i - T_i) + \varepsilon_{p,i} = \frac{(x_i - x^0)}{\rho_i^0}\delta x_i + \frac{(y_i - y^0)}{\rho_i^0}\delta y_i + \frac{(z_i - z^0)}{\rho_i^0}\delta z_i - c\delta(\delta t_i) \tag{64.7}$$

where $\rho_i^0$ is the pseudorange of predict position.

Define the following arguments:

$$\begin{gathered} \delta\rho_i = \rho_i^0 - (\rho_i - I_i - T_i) \\ a_{xi} = \frac{x_i - x^0}{\rho_i^0} \;\; a_{yi} = \frac{y_i - y^0}{\rho_i^0} \;\; a_{zi} = \frac{z_i - z^0}{\rho_i^0} \end{gathered} \tag{64.8}$$

Ignoring the measurement noise, the formula (64.7) simplifies:

$$\delta\rho_i = a_{xi}\delta x_i + a_{yi}\delta y_i + a_{zi}\delta z_i - c\delta(\delta t_i) \tag{64.9}$$

When the available satellites (N) are more than four satellites, the equations list as:

$$\delta\vec{\rho}_i = H_{pp}\delta\vec{x}_i \tag{64.10}$$

where:

$$\delta\vec{\rho}_i = \begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \\ \vdots \\ \delta\rho_N \end{bmatrix} \quad \delta\vec{x} = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \\ -c\delta(\delta t) \end{bmatrix} \quad H_{pp} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ a_{x3} & a_{y3} & a_{z3} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{xN} & a_{yN} & a_{zN} & 1 \end{bmatrix} (N \geq 4) \tag{64.11}$$

When $H^T H$ is symmetric, positive definiteness and reversible, the solution of the problem is given by:

$$\delta\vec{x} = (H_{pp}^T H_{pp})^{-1} H_{pp}^T \delta\vec{\rho}_i \tag{64.12}$$

From the Eq. (64.12), the offset of user position $(x, y, z, \delta t)$ and predict position $(x^0, y^0, z^0, \delta t^0)$ is given. If the offset is small, the user position is get. If the offset is large, the iteration is used to lessen the offset.

### *64.3.3 Doppler-Aided Position Algorithm*

In signal transmission process, the doppler measurement is not sensitive to environment noise. So, the doppler measurement is more accuracy than that of pseudorange and thus can be used to assist smooth positioning.

Similar with formula (64.4), linear Eq. (64.6) at the predicted position and velocity $(x^0, y^0, z^0, \delta t^0, v_x^0, v_y^0, v_z^0)$ [5]:

$$\begin{aligned} f_{di} = \frac{f^0}{c} \Bigg\{ & \left( \frac{(v_{xi} - v_x^0)}{\rho_i^0} + \frac{(x_i - x^0)}{(\rho_i^0)^3} \right) \delta x + \left( \frac{(v_{yi} - v_y^0)}{\rho_i^0} + \frac{(y_i - y^0)}{(\rho_i^0)^3} \right) \delta y \\ & + \left( \frac{(v_{zi} - v_z^0)}{\rho_i^0} + \frac{(z_i - z^0)}{(\rho_i^0)^3} \right) \delta z + \frac{(x_i - x^0)}{\rho_i^0} \delta v_x \\ & + \frac{(y_i - y^0)}{\rho_i^0} \delta v_y + \frac{(z_i - z^0)}{\rho_i^0} \delta v_z \Bigg\} + \delta f \end{aligned} \tag{64.13}$$

where $\delta f$ is frequency deviation

Define the following argument:

$$b_{xi} = \frac{f^0}{c} \left( \frac{(v_{xi} - v_x^0)}{\rho_i^0} + \frac{(x_i - x^0)}{(\rho_i^0)^3} \right) \qquad c_{vxi} = \frac{f^0}{c} \left( \frac{(x_i - x^0)}{\rho_i^0} \right)$$

$$b_{yi} = \frac{f^0}{c} \left( \frac{(v_{yi} - v_y^0)}{\rho_i^0} + \frac{(y_i - y^0)}{(\rho_i^0)^3} \right) \qquad c_{vyi} = \frac{f^0}{c} \left( \frac{(y_i - y^0)}{\rho_i^0} \right)$$

$$b_{zi} = \frac{f^0}{c}\left(\frac{(v_{zi} - v_z^0)}{\rho_i^0} + \frac{(z_i - z^0)}{(\rho_i^0)^3}\right) \qquad c_{vzi} = \frac{f^0}{c}\left(\frac{(z_i - z^0)}{\rho_i^0}\right) \tag{64.14}$$

The formula (64.13) simplifies:

$$f_{di} = b_{xi}\delta x + b_{yi}\delta y + b_{zi}\delta z + c_{vxi}\delta v_x + c_{vyi}\delta v_y + c_{vzi}\delta v_z + \delta f \tag{64.15}$$

In (64.15), there are seven unknowns which mean that seven satellites measurements are used. However, in urban, it is not common to get seven satellites. Combing (64.10) and (64.15), the position and velocity are solved with four satellites.

$$\begin{bmatrix} \delta\vec{\rho} \\ f_d \end{bmatrix} = \begin{bmatrix} H_{pp} & 0 \\ H_{pf} & H_{ff} \end{bmatrix} \begin{bmatrix} \delta\vec{x} \\ \delta\vec{v} \end{bmatrix} \tag{64.16}$$

where:

$$\begin{aligned} f_d &= \begin{bmatrix} f_{d1} \\ f_{d2} \\ f_{d3} \\ \vdots \\ f_{dN} \end{bmatrix} H_{ff} = \begin{bmatrix} c_{vx1} & c_{vy1} & c_{vz1} & 1 \\ c_{vx2} & c_{vy2} & c_{vz2} & 1 \\ c_{vx3} & c_{vy3} & c_{vz3} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ c_{vxN} & c_{vyN} & c_{vzN} & 1 \end{bmatrix} \\ \delta\vec{v} &= \begin{bmatrix} \delta v_x \\ \delta v_y \\ \delta v_z \\ \delta f \end{bmatrix} H_{pf} = \begin{bmatrix} b_{x1} & b_{y1} & b_{z1} & 0 \\ b_{x2} & b_{y2} & b_{z2} & 0 \\ b_{x3} & b_{y3} & b_{z3} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ b_{xN} & b_{yN} & b_{zN} & 0 \end{bmatrix} \end{aligned} \tag{64.17}$$

Similar, the unbiased estimate of $\delta x, \delta y, \delta z, \delta t, \delta v_x, \delta v_y, \delta v_z, \delta f$ is given:

$$\begin{bmatrix} \delta\vec{x} \\ \delta\vec{v} \end{bmatrix} = (H^T H)^{-1} H^T \begin{bmatrix} \delta\vec{\rho} \\ f_d \end{bmatrix} \tag{64.18}$$

where:

$$H = \begin{bmatrix} H_{pp} & 0 \\ H_{pf} & H_{ff} \end{bmatrix} \tag{64.19}$$

The result of the Eq. (64.18) is

$$\begin{bmatrix} \delta\vec{x} \\ \delta\vec{v} \end{bmatrix} = (H^T W H)^{-1} H^T W \begin{bmatrix} \delta\vec{\rho} \\ f_d \end{bmatrix} \tag{64.20}$$

where the weight is defined as:

$$W = \begin{bmatrix} \left[\sigma_{\rho_i}^{-2}\right] & 0 \\ 0 & \left[\sigma_{f_i}^{-2}\right] \end{bmatrix} \tag{64.21}$$

$\sigma$ is the variance of measurement

### *64.3.4 Optimize Loop Parameter*

In order to improve the accuracy of the Doppler measurement, the noise bandwidth is reduced which reduce the noise for the carrier loop. The noise bandwidth of the control into the loop filter the noise energy, the effect can be improved by reducing the noise bandwidth of the loop filter, to improve tracking accuracy.

The width of the noise bandwidth depends on the tracking dynamic and SNR. Small noise bandwidth may reduce the noise in the band, but not be able to track a large dynamic, it is easy to cause the frequency and phase tracking error; large noise bandwidth will introduce a large noise energy, affecting the the signal tracking stability [6].

Users generally smaller dynamic system dynamics is mainly caused by the satellite movement. Beidou system system uses a three-orbit satellites, the dynamic of the GEO satellites is very low. For this reason, we can reduce the noise bandwidth of the GEO satellite tracking filter to make it match with MEO and IGSO, the same dynamic, improved the GEO carrier ring accuracy so without reducing the dynamic case.

Considering the essential requirement PLL loop parameter:

$$\sqrt{\sigma_{tPLL}^2 + \sigma_v^2 + \sigma_A^2} + \frac{\theta_e}{3} \le 15^\circ \tag{64.22}$$

where $\sigma_{tPLL}$ is $1\sigma$ thermal noise, $\sigma_v$ is $1\sigma$ vibration, $\sigma_A$ is Allan deviation, $\theta_e$ is PLL dynamic stress.

Consider thermal noise errors and dynamic stress error, the state of motion of the vehicle terminal user as example [7], we can set the PLL noise bandwidth is 15 Hz for GEO and 30 Hz for others respectively. The accuracy of position result is improved by improving the GEO doppler measurement accuracy and weight.

## 64.4 Testing and Analysis

To demonstrate the performance, the proposed algorithm has been investigated using Beidou date under static real-time condition. In this section, the detail of experiments is introduced.

The platform (Fig. 64.2) is built with GPS/BD2 chip named BD-ZS3121 which is designed by lab of SoC, Peking University.

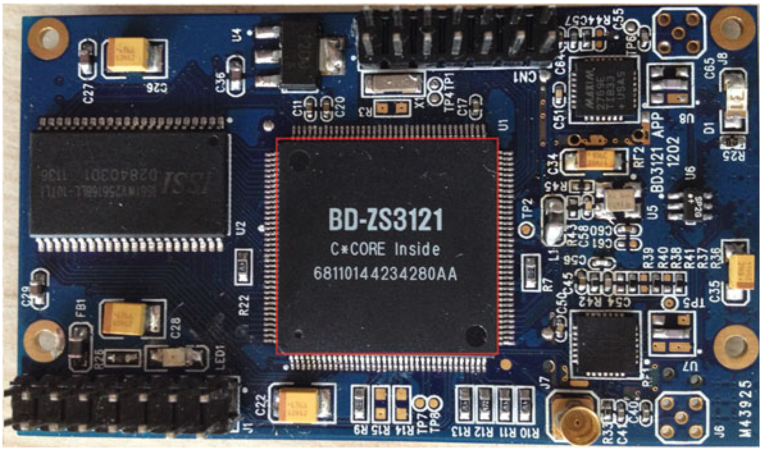


**Fig.. 64.2** BD-ZS3121 Platform

In order to compare different algorithms, the pseudorange algorithm, Hatch filter and dopper-aided position algorithm. During the entire measurement campaign, six Beidou satellites (four GEO satellites and two MEO satellites) were captured with a cut-off elevation angel of $10^{\circ}$. Time interval of an epoch is 1 s, and 1,000 continuous points are collected for each method. The signal power is −130 dBm. The means and variances are shown in Tables 64.1 and 64.2.

From Tables 64.1 and 64.2, the carrier phase smoothed or doppler smoothed pseudorange reduce the variances and mean. When the probability of cycle slip is in low level, the Hatch filter provide more accuracy than doppler-aided method but the different is not evident.

In order to further verify the cycle slip effect on smoothing progress, additional tests are included in which the signal power is attenuated extra 10 dB (Tables 64.3, 64.4).

**Table 64.1** Test result of user position (−130dBm)

| Method | Mean | Variance | Maximum |
|---|---|---|---|
| Pseudorange | 4.27 | 4.50 | 10.42 |
| Hatch filter | 3.17 | 3.09 | 4.38 |
| Doppler-aided | 3.20 | 3.22 | 4.16 |

**Table 64.2** Test result of user velocity (−130dBm)

| Method | Mean | Variance | Maximum |
|---|---|---|---|
| Pseudorange | 0.028 | 0.029 | 0.075 |
| Hatch filter | 0.028 | 0.020 | 0.079 |
| Doppler-aided | 0.027 | 0.021 | 0.060 |

**Table 64.3** Test result of user position (−140dBm)

| Method | Mean | Variance | Maximum |
|---|---|---|---|
| Pseudorange | 9.38 | 10.4385 | 28.33 |
| Hatch filter | 7.91 | 5.0442 | 15.58 |
| Doppler-aided | 5.91 | 4.0709 | 7.47 |

**Table 64.4** Test result of user velocity(−140dBm)

| Method | Mean | Variance | Maximum |
|---|---|---|---|
| Pseudorange | 0.065 | 0.0838 | 0.127 |
| Hatch filter | 0.038 | 0.0658 | 0.104 |
| Doppler-aided | 0.040 | 0.0417 | 0.095 |

In Tables 64.3 and 64.4, the variance of Hatch filter immensely increases with the increasing cycle slip times. In each time of cycle slip, the Hatch filter accumulates error. While, the doppler-aided algorithm is not sensitive to cycle slip and the accuracy is high than Hatch filter.

From the test result, in low signal-to-noise ratio environment, due to the probable of the loss of lock, the position accuracy of Hatch filter is fall. Doppler-aided position algorithm provides a good robustness, and low noise bandwidth improves the accuracy of the measurement data, and achieves a good position results.

## 64.5 Conclusions

The Doppler-assisted positioning algorithm is introduced. This algorithm combines both pseudorange and doppler measurement. Taking advantage of the low dynamic satellites, the PLL noise bandwidth is lesson to improve the measurement accuracy. Under low signal-to-noise ratio, comparing with Hatch filter, doppler measurement is not suffer from cycle slips and improve the performance and stability.

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